Oblig2 FYS9130

Deadline: Tuesday 14/9 at 12.15 (beginning of class)

1. **Covariant derivatives of the metric is zero by definition**

$$g_{\mu\nu;\alpha} = 0 \tag{1}$$

show that this is so by using the definition of the covariant derivative in terms of Christoffel symboles, and the definition of Christoffel symbols in terms of the metric.

2. **Integration by parts**

a) Use the definiton of the covariant derivative in terms of Christoffelsymbols to show that the product rule holds also for covariant derivatives:

$$(A^\mu B_\mu)_{;\nu} = A^\mu{}_{;\nu} B_\mu + A^\mu B_{\mu;\nu} \tag{2}$$

b) Show that

$$\Gamma^\mu{}_{\mu\nu} = \frac{1}{\sqrt{-g}}(\sqrt{-g})_{,\nu} \tag{3}$$

c) Further show that

$$A^\mu{}_{;\mu} = \frac{1}{\sqrt{-g}}(\sqrt{-g}A^\mu)_{,\mu} \tag{4}$$

d) By using the normal Stoke's theorem, show that

$$\int d^n x \sqrt{-g} \nabla_\mu A^\mu \tag{5}$$

equals a surface term

e) You do not need to answer this question. Just note that when ignoring a surface term we have

$$\int d^n x \sqrt{-g} A^\mu B_{;\mu} = -\int d^n x \sqrt{-g} A^\mu{}_{;\mu} B \tag{6}$$

3. **Variation of action**

An action for gravity non-minimally coupled to a scalar field is given by:

$$S = \int d^n x \sqrt{-g} \left(\frac{M_P^2}{2} R - \frac{\xi}{2} R\phi^2 - \frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - V(\phi) \right) \tag{7}$$

where M_P^2 and ξ are constants.

a) Variate this action with respect to the metric $g_{\mu\nu}$ and find the corresponding equations of motion.

b) Variate this action with respect to the scalar field ϕ and find the corresponding equation of motion.

If possible, deliver a paper copy, handwritten is ok. Otherwise, e-mail to ingunnkw@fys.uio.no