Oblig4 FYS9130

Deadline: Tuesday 28/9 at 12.15 (beginning of class)

1. **4+n split metric**

   Assume a $D = 4 + n$ dimensional spacetime split into our usual 4-dimensional spacetime times an n-dimensional compact space:

   $$ds^2 = \bar{g}_{MN} dX^M dX^N = \hat{g}_{\mu\nu}(x) dx^\mu dx^\nu + b^2(x) \tilde{\gamma}_{ij}(y) dy^i dy^j \,. \tag{1}$$

   Here the D coordinates $\{X^M\} = \{x^\mu, y^i\}$. The 4-dimensional geometry, described by the metric $g_{\mu\nu}$, is left general, while the extra n-dimensional space is described by a scale factor $b(x^\mu)$, times an internal metric $\tilde{\gamma}_{ij}$ for the extra dimensions. Quantities in $D = 4 + n$ dimensions are indicated by bars and upper-case Latin indices, and quantities in the extra n dimensions by tildes and lower-case Latin indices. Greek indices and hats indicate quantities in the four-dimensional spacetime. Note that we have

   $$\hat{g}_{\mu\nu,i} = 0 \tag{2}$$
   $$b_{,i} = 0 \tag{3}$$
   $$\tilde{\gamma}_{ij,\mu} = 0 \tag{4}$$

   a) Show that
   $$\sqrt{-\bar{g}} = \sqrt{-\hat{g}} \sqrt{\tilde{\gamma}}\, b^n \,. \tag{5}$$

   b) Calculate the D-dimensional Riemann tensor components $\bar{R}^M_{\ \ NPQ}$ in terms of the 4-dimensional $\hat{R}^\mu{}_{\nu\rho\sigma}$, the n-dimensional $\tilde{R}^i{}_{jkl}$ for the internal geometry and the scale factor b.

   c) Do the same for the Ricci tensor

   d) Show that for the Ricci scalar we have

   $$\bar{R} = \hat{R} + \frac{\tilde{R}}{b^2} - \frac{2n}{b} \hat{g}^{\rho\sigma} \hat{\nabla}_\rho \hat{\nabla}_\sigma b - \frac{n(n-1)}{b^2} \hat{g}^{\rho\sigma} \hat{\nabla}_\rho b \hat{\nabla}_\sigma b \,. \tag{6}$$

2. **Kaluza-Klein theory**

   Assume pure GR in D dimensions (α being a constant)

   $$S = \int d^{\,4+n} X \sqrt{-\bar{g}} \alpha \bar{R} \tag{7}$$

   and that the metric is split into $4 + n$ dimensions as given above. Also assume that the internal metric for the extra dimensions is maximal symmetric. Perform the integral over the n extra dimesions to arrive at a four-dimensional theory. You can define $\mathcal{V}_i$ to be the internal volume of the extra-dimensional space:

   $$\mathcal{V}_i = \int d^{\,n} y \sqrt{\tilde{\gamma}} \tag{8}$$

If possible, deliver a paper copy, handwritten is ok. Otherwise, e-mail to ingunnkw@fys.uio.no