Oblig5 FYS9130

Deadline: Tuesday 12/10 at 12.15 (beginning of class)

1. **Amendola, Polarski and Tsujikawa: Are $f(R)$ dark energy models cosmologically viable?**

   Assume an action given as eq.1 in APT:

   $$S = \int d^n x \sqrt{-g} \left(\frac{M_P^2}{2} f(R) + \mathcal{L}_m \right) \tag{1}$$

   a) Assuming $f(R) = R^p$ and defining $F = f'(R)$, show that we can write

   $$S = \int d^n x \sqrt{-g} \left(\frac{M_P^2}{2p} F R + \mathcal{L}_m \right) \tag{2}$$

   b) By performing a Weyl transformation $g_{\mu\nu} = \Omega^2(F) \tilde{g_{\mu\nu}}$ and redefining a scalar field $\phi = \phi(F)$, show that the above action can be transformed into an Einstein frame action on the form given by eq.3 in APT:

   $$S = \int d^n x \sqrt{-\tilde{g}} \left(\frac{M_P^2}{2} \tilde{R} - \frac{1}{2} \tilde{g}^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - V(\phi) + \tilde{\mathcal{L}}_m(\phi) \right) \tag{3}$$

   Which functions $\Omega^2(F)$ and $\phi(F)$ are needed? You can take advantage of the known Weyl transformation of the Ricci scalar in 4 dimensions:

   $$R = \Omega^{-2} \tilde{R} - 6 \Omega^{-3} \tilde{\Box} \Omega \tag{4}$$

   c) What are the results you get for $V(\phi)$ and $\tilde{\mathcal{L}}_m(\phi)$?

   d) Find the equations of motion for the Einstein frame action. You can assume that the Jordan frame matter Lagrangian $\mathcal{L}_m$ describes a perfect fluid with $T^\mu_{\ \nu} = \mathbf{diag}(-\rho_m, p_m, p_m, p_m)$

   e) Assume that the Einstein frame metric $\tilde{g_{\mu\nu}}$ is a flat Robertson-Walker metric. Compare your equations of motion with eqs. 5 and 6 in APT. What is your value of β?

   f) APT claim that they will have $\beta = \frac{1}{2}$ regardless of the form of $f(R)$. Do you believe them?

If possible, deliver a paper copy, handwritten is ok. Otherwise, e-mail to ingunnkw@fys.uio.no