Oblig6 FYS9130

Deadline: Thursday 21/10 at 14.15 (beginning of class)

1. **Double exponential potential and cosmological scaling solutions**

   Start with the standard Friedmann equations

$$H^2 = \frac{1}{3M^2}\left(\rho_{tot}\right) \tag{1}$$

$$2\dot{H} = -\frac{1}{M^2}\left(\rho_{tot} + p_{tot}\right) \tag{2}$$

   Assume an universe filled with a non-relativistic perfect fluid ("matter", $p_m = 0$), radiation ($p_r = \frac{1}{3}\rho_r$) and a scalar field ($\rho_\phi = \rho_k + \rho_{p1} + \rho_{p2}$, $\rho_k = \frac{1}{2}\dot{\phi}^2$, $\rho_{p1} = Ae^{-\frac{\lambda}{M}\phi}$, $\rho_{p2} = Be^{-\frac{\gamma}{M}\phi}$, $p_k = \rho_k$, and $p_{px} = -\rho_{px}$). That is, the scalar field potential is

$$V(\phi) = Ae^{-\frac{\lambda}{M}\phi} + Be^{-\frac{\gamma}{M}\phi} = \rho_{p1} + \rho_{p2} \tag{3}$$

   Assume no extra couplings between matter and radiation, between matter and ϕ or between radiation and ϕ, so we have:

$$\dot{\rho}_m = -3H(\rho_m + p_m) \tag{4}$$

$$\dot{\rho}_r = -3H(\rho_r + p_r) \tag{5}$$

$$\dot{\rho}_\phi = -3H(\rho_\phi + p_\phi) \tag{6}$$

   a) Show that we have

$$\dot{\rho}_m = -3H\rho_m \tag{7}$$

$$\dot{\rho}_r = -4H\rho_r \tag{8}$$

$$\dot{\rho}_{p1} = -\sqrt{6}\lambda H\rho_{p1}\sqrt{\Omega_k} \tag{9}$$

$$\dot{\rho}_{p2} = -\sqrt{6}\gamma H\rho_{p2}\sqrt{\Omega_k} \tag{10}$$

$$\dot{\rho}_k = -6H\rho_k - \dot{\rho}_{p1} - \dot{\rho}_{p2} \tag{11}$$

   b) Find the derivative of the relative densities with respect to the natural logarithm of the scale factor (as a function of Ω's, β and γ). Show that these can be written

$$\Omega'_m = -3\Omega_m + 3\Omega_m\left(\Omega_m + \frac{4}{3}\Omega_r + 2\Omega_k\right) \tag{12}$$

$$\Omega'_r = -4\Omega_r + 3\Omega_r\left(\Omega_m + \frac{4}{3}\Omega_r + 2\Omega_k\right) \tag{13}$$

$$\Omega'_k = -6\Omega_k + \sqrt{6}\lambda\Omega_{p1}\sqrt{\Omega_k} + \sqrt{6}\gamma\Omega_{p2}\sqrt{\Omega_k} + 3\Omega_k\left(\Omega_m + \frac{4}{3}\Omega_r + 2\Omega_k\right) \tag{14}$$

$$\Omega'_{p1} = -\sqrt{6}\lambda\Omega_{p1}\sqrt{\Omega_k} + 3\Omega_{p1}\left(\Omega_m + \frac{4}{3}\Omega_r + 2\Omega_k\right) \tag{15}$$

$$\Omega'_{p2} = -\sqrt{6}\gamma\Omega_{p2}\sqrt{\Omega_k} + 3\Omega_{p2}\left(\Omega_m + \frac{4}{3}\Omega_r + 2\Omega_k\right) \tag{16}$$

c) Find the fixpoints of the above equation set.

If possible, deliver a paper copy, handwritten is ok. Otherwise, e-mail to ingunnkw@fys.uio.no