Oblig8 FYS9130

Deadline: Thursday 4/11 at 14.15 (beginning of class)

1. **A cosmological constant**

   Assume an action including a cosmological constant Λ

   $$S = \int d^4x \sqrt{-g}\left[\frac{M_P^2}{2}(R - 2\Lambda) + \mathcal{L}_m\right] \tag{1}$$

   a) Show by variation of the action that the equations of motion can be written
   $$M_P^2 E_{\mu\nu} = T^m_{\mu\nu} + T^\Lambda_{\mu\nu} \tag{2}$$
   where $T^\Lambda_{\mu\nu}$ is the energy-momentum tensor for a perfect fluid with $\rho_\Lambda + p_\Lambda = 0$. Find ρ_Λ and p_Λ.

   b) Acoording to WMAP's 7-year release, the best fit cosmological parameters values for a ΛCDM model includes $\Omega_{\Lambda 0} = 0.734$ for the relative cosmological constant energy density today, as well as $H_0 = 71.0$ km/s/Mpc for the value of the Hubble parameter today. Find the corresponding value of $\rho_{\Lambda 0}$ expressed in eV.

2. **The cosmological constant as a quantum mechanical vacuum energy**

   Let us assume that some quantum mechanical vaccum energy density $\epsilon_0 = E_0/V$ is inserted into the action as a matter contribution, $\mathcal{L}_\epsilon = -\epsilon_0$, and

   $$S = \int d^4x \sqrt{-g}\left[\frac{M_P^2}{2}R + \mathcal{L}_m + \mathcal{L}_\epsilon\right] \tag{3}$$

   a) Show that this corresponds to a cosmological constant. Find Λ and ρ_Λ as a function of ϵ_0.

   b) Let ϵ_0 be the vacuum energy density of a free, massless, scalar field regularized with a Planck-scale cutoff M_P. Find the corresponding ρ_Λ expressed in eV. Compare this with the measured value of $\rho_{\Lambda 0}$ calculated above.

If possible, deliver a paper copy, handwritten is ok. Otherwise, e-mail to ingunnkw@fys.uio.no