Oblig9 FYS9130

Deadline: Tuesday 16/11 at 12.15 (beginning of class)

1. **Weyl transformations**

   A transformation of a D-dimensional metric is given by

   $$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \tag{1}$$

   where $\Omega = \Omega(x^\mu)$. Show that we have

   $$\tilde{\Box}\phi = \Omega^{-2}\Box\phi + (D-2)\Omega^{-3}g^{\mu\nu}\Omega_{,\mu}\phi_{,\nu} \tag{2}$$

2. **Vacuum energy for gravitons**

   Assume only gravity in $D = 1+3+1$ dimensions, where the last dimension is compact of size L. Interpret this as having gravitons in a flat background and calculate the vacuum energy density ϵ_0 of the gravitons. This can be done by finding the vacuum energy for a scalar field, and multiply with the degrees of freedom ("polarisations") for the graviton. The vacuum energy density $\epsilon_0 = \epsilon_M + \epsilon_C$ consists of an infinite part ϵ_M and a finite Casimir part ϵ_C due to the compact dimension. Use a Planck-mass cutoff to regularize the infinite part ϵ_M, and find ϵ_0.

3. **Kaluza Klein reduction**

   Let us assume that the above vacuum energy density ϵ_0 is inserted into the action as a matter contribution, $\bar{\mathcal{L}}_\epsilon = -\epsilon_0$:

   $$S = \int d^5\bar{X}\sqrt{-\bar{g}}\left[\frac{\bar{M}^3}{2}\bar{R} + \bar{\mathcal{L}}_\epsilon\right] \tag{3}$$

   Let us also assume a Kaluza-Klein split of our spacetime

   $$ds^2 = \bar{g}_{MN}dX^M dX^N = \hat{g}_{\mu\nu}(x)dx^\mu dx^\nu + b^2(x)dy^2. \tag{4}$$

   with notation as in oblig4.

   a) Show that when integrating out the extra dimension we get a 4-dimensional action on the form:

   $$S = \int d^4x\sqrt{-\hat{g}}\, b\Big[\frac{M_P^2}{2}\left(\hat{R} - \frac{2}{b}\hat{\Box}b\right) - \frac{\hat{\epsilon}}{b^p} - \frac{\hat{\sigma}}{b^q}\Big] \tag{5}$$

   where $\hat{\epsilon}$, $\hat{\sigma}$, p and q are constants.

   b) Show that after a Weyl transformation of the metric $\hat{g_{\mu\nu}} = \Omega^2 g_{\mu\nu}$ and a redefinition of the scalar field $b \to \phi(b)$ we have the Einstin frame action

   $$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}\phi_{,\mu}\phi^{,\mu} - V(\phi)\right] \tag{6}$$

   where

   $$V(\phi) = Ae^{-\frac{\lambda}{M}\phi} + Be^{-\frac{\gamma}{M}\phi} \tag{7}$$

   Find the constants A, B, γ, and λ.

If possible, deliver a paper copy, handwritten is ok. Otherwise, e-mail to ingunnkw@fys.uio.no