Problem set 2 FYS9130 – Week 36

1. **Robertson-Walker metric with spatial curvature**

   Assume a metric given by the Robertson-Walker line element $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right) \quad (1)$$

   a) Find the non-zero Christoffel symbols

   b) Find the non-zero components of the Riemann tensor

   c) Find the non-zero components of the Ricci tensor

   d) Calculate the Ricci scalar

   c) Find the non-zero components of the Einstein tensor

2. **Friedmann equations**

   Assume that the geometry of our universe is described with the above metric, and that the matter content of the universe can be described as a perfect fluid, parametrised only with its pressure p and energy density ρ

$$T_{\mu\nu} = (\rho + p)U_\mu U_\nu + p g_{\mu\nu} \quad (2)$$

   where the 4-velocity $U^\mu = [1, 0, 0, 0]$ in a comoving coordinate system.

   a) Write down the components of the Einstein equations for this system. These are the Friedmann equations.

   b) Assume a constant ratio between the pressure and the energy density, $p = w\rho$. Solve the Friedmann equations to find $a(t)$, $\rho(a)$ and $\rho(t)$.

   c) What are the constraint on w for the universe to experience positive acceleration, $\ddot{a} > 0$?