Problem set 3 FYS9130 – Week 37-38

1. **Weyl transformations**

   A transformation of a D-dimensional metric is given by

   $$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \tag{1}$$

   where $\Omega = \Omega(x^\mu)$. Show that we have the following relationships between tilded and untilded quantities:

   a) The determinant of the metric

   $$\sqrt{-\tilde{g}} = \Omega^D \sqrt{-g} \tag{2}$$

   b) The Riemann tensor

   $$\begin{aligned}\tilde{R}_{\alpha\mu\beta\nu} =& \Omega^2 R_{\alpha\mu\beta\nu} + \Omega\Big(g_{\alpha\nu}\Omega_{;\mu\beta} + g_{\mu\beta}\Omega_{;\alpha\nu} - g_{\alpha\beta}\Omega_{;\mu\nu} - g_{\mu\nu}\Omega_{;\alpha\beta}\Big) \\ &+ 2\Big(g_{\alpha\beta}\Omega_{,\mu}\Omega_{,\nu} + g_{\mu\nu}\Omega_{,\alpha}\Omega_{,\beta} - g_{\alpha\nu}\Omega_{,\mu}\Omega_{,\beta} - g_{\mu\beta}\Omega_{,\alpha}\Omega_{,\nu}\Big) \\ &+ \Big(g_{\alpha\nu}g_{\mu\beta} - g_{\alpha\beta}g_{\mu\nu}\Big)\Omega_{,\sigma}\Omega^{,\sigma}\end{aligned} \tag{3}$$

   c) Ricci tensor

   $$\begin{aligned}\tilde{R}_{\mu\nu} =& R_{\mu\nu} - \Omega^{-1}\Big[(D-2)\Omega_{;\mu\nu} + g_{\mu\nu}\Box\Omega\Big] \\ &+ \Omega^{-2}\Big[2(D-2)\Omega_{,\mu}\Omega_{,\nu} - (D-3)g_{\mu\nu}\Omega_{,\sigma}\Omega^{,\sigma}\Big]\end{aligned} \tag{4}$$

   d) Ricci scalar

   $$\tilde{R} = \Omega^{-2}R - 2(D-1)\Omega^{-3}\Box\Omega - (D-1)(D-4)\Omega^{-4}\Omega_{,\sigma}\Omega^{,\sigma} \tag{5}$$

   e) Finally, calculate $\phi_{\tilde{;}\mu\nu}$ and $\tilde{\Box}\phi$ in terms of untilded metrics and derivatives.