Problem set 5 FYS9130 – Week 40

1. **Cosmological scaling solutions**

   Start with the standard Friedmann equations

$$H^2 = \frac{1}{3M^2}\left(\rho_{tot}\right) \tag{1}$$

$$2\dot{H} = -\frac{1}{M^2}\left(\rho_{tot} + p_{tot}\right) \tag{2}$$

   Assume an universe filled with a non-relativistic perfect fluid ("matter", $p_m = 0$) coupled to a scalar field ($\rho_\phi = \rho_k + \rho_p$, $p_k = \rho_k$, and $p_p = -\rho_p$) with coupling β:

$$\dot{\rho}_\phi = \dot{\rho}_k + \dot{\rho}_p = -6H\rho_k + 2\beta H\rho_m\sqrt{\Omega}_k \tag{3}$$

$$\dot{\rho}_m = -3H\rho_m - 2\beta H\rho_m\sqrt{\Omega}_k \tag{4}$$

   For

$$\rho_p = V(\phi) = Ae^{-\frac{\lambda}{M}\phi} \tag{5}$$

   we also have

$$\dot{\rho}_p = -\sqrt{6}\lambda H\rho_p\sqrt{\Omega}_k \tag{6}$$

   a) Find the derivative of the relative densities with respect to the natural logharitm of the scale factor (as a function of Ω's, β and λ).

   b) Find the fixpoints of the above equation set.

   c) Are there any stable fixpoints?