Problem set 6 FYS9130 – Week 41

1. **Solid angles**

   Start with the Gaussian integral in n dimensions:

$$I_n = \int_{-\infty}^{\infty} d^n k e^{-k^2} \tag{1}$$

   a) Show that in cartesian coordinates we find

$$I_n = \int_{-\infty}^{\infty} dk_1 \int_{-\infty}^{\infty} dk_2 \ \ldots \int_{-\infty}^{\infty} dk_n e^{-({k_1}^2+{k_2}^2+\cdots+{k_n}^2)} = \pi^{\frac{n}{2}} \tag{2}$$

   b) Show that in spherical coordinates we find

$$I_n = \int_0^{\infty} \Omega_{n-1} k^{n-1} dk e^{-k^2} = \Omega_{n-1}\frac{1}{2}\Gamma(\frac{n}{2}) \tag{3}$$

   Where the solid angle (romvinkelelementet) Ω_{n-1} in $n-1$ dimensions is the $(n-1)$-dimensional surface of a unit sphere in n dimensions.

   c) Use this to find the general expression for the solid angle in arbitrary dimensions

2. **The Gamma function**

   Use the definition of the Gamma function:

$$\Gamma(n) = \int_0^{\infty} dx\, x^{n-1} e^{-x} \tag{4}$$

   to show that

$$\frac{1}{a^n} = \frac{1}{\Gamma(n)} \int_0^{\infty} dx\, x^{n-1} e^{-ax} \tag{5}$$

   Further show that

$$\int \frac{d^d p}{(2\pi)^d} \frac{1}{(p^2+m^2)^N} = (4\pi)^{-\frac{d}{2}} \frac{\Gamma(N-\frac{d}{2})}{\Gamma(N)} (m^2)^{\frac{d}{2}-N} \tag{6}$$